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T.Y.B.Sc. (Sem. VI) Examination March - 2026

Mathematics Major - MH - MJ - 603 - Real Analysis - II

[Total Marks: 25

સૂચના : / Instructions	Seat No.:
(1) ઉપરોક્ત દર્શાવેલ વિગતો ઉત્તરવહી પર અવશ્ય લખવી. Fill up strictly the above details on your answer book (2) All questions are compulsory. (3) Figures to the right indicate marks of the corresponding question.	Student's Signature

Q:1 Answer the following. (Any Five) **05**

1 Define: Bounded Sequence.

2 State D'Alembert's Ratio test for convergence of series.

3 Give an example of a sequence $\{s_n\}_{n=1}^{\infty}$ which is not bounded but $\lim_{n\to\infty} \frac{s_n}{n} = 0$

4 Define alternating series with illustration.

5 Find $\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^{n+1}$

6 Check the convergence of a series $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} n$

Q:2 Answer the following. (Any Two) **10**

1 Show that any bounded sequence of real numbers has a convergent subsequence.

2 If $\{s_n\}_{n=1}^{\infty}$ is a Cauchy sequence of real numbers which has a subsequence converging to L, then prove that $\{s_n\}_{n=1}^{\infty}$ also converges to L.

3 Define: Non-increasing Sequence. Also show that $S_n = \frac{2\cdot4\cdot6\cdot\ldots\cdot2n}{1\cdot3\cdot5\cdot\ldots\cdot(2n-1)}\frac{1}{n^2}$ is non-increasing.

Q:3 Answer the following. (Any Two) **10**

1 If the series $\sum_{n=1}^{\infty} a_n$ converges to A and the series $\sum_{n=1}^{\infty} b_n$ converges to B, then prove the following:

(a) $\sum_{n=1}^{\infty}(a_n + b_n)$ converges to A+B (b) $\sum_{n=1}^{\infty} ca_n$ converges to c A.

2 Prove the following:

(a) If $0 < x < 1$ then $\sum_{n=1}^{\infty} x^n$ converges to $\frac{1}{1-x}$. (b) If $x \geq 1$ then $\sum_{n=1}^{\infty} x^n$ diverges.

3 Check the convergence of :

(a) $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{2^n}$ (b) $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty}\frac{2n}{n^2-4n+7}$